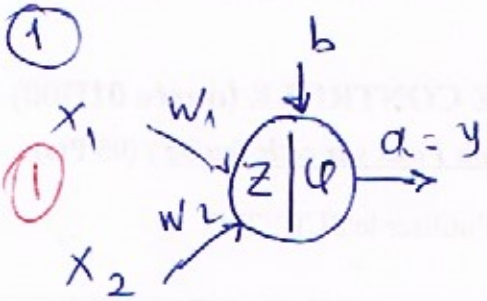
Contrôle UEM32 (O.I.A) M2 MAI 23/24

Corrigé Type

EXERCICE (1)

1°/



x_1	x_2	y_d
1	1	0
-1	1	0
-3	0	1
-2	-2	1

2°/ $* z = w_1 x_1 + w_2 x_2 + b$

$$* \varphi(z) = \begin{cases} 1 & z \geq 0 \\ 0 & \text{ailleurs} \end{cases}$$ (1K)

$* a = \varphi(z) = y$

$w_1^0 = -1$
$w_2^0 = 0,5$
$b^0 = 0,5$
$\alpha = 1$

3°/ Selon la Regle de Hebb

$$w(k+1) = w(k) - \alpha(a - y_d)$$
$$b(k+1) = b(k) - \alpha(a - y_d)$$

Ite 0 : $* x_1 = 1, x_2 = 1, y_d = 0$

- $z_1^0 = -1(+1) + 0,5 \cdot 1 + 0,5 = 0 \geq 0 \Rightarrow a = 1 \neq y_d \Rightarrow$ MAJ
- $w_1^1 = -1 - 1(1-0) = -2$
  $w_2^1 = 0,5 - 1(1-0) = -0,5$
  $b^1 = 0,5 - 1(1-0) = -0,5$ (1)

** $x_1 = -1, x_2 = 1, y_d = 0$

- $z_2 = -2(-1) - 0,5(1) - 0,5 = 1 > 0 \Rightarrow a = 1 \neq y_d \Rightarrow$ MAJ
- $w_1^2 = -2 - 1(1-0) = -3$
  $w_2^2 = -0,5 - 1(1-0) = -1,5$
  $b^2 = -0,5 - 1(1-0) = -1,5$

* $x_1 = -3, x_2 = 0, y_d = 1$

• $z_3^0 = -3(-3) - 1{,}5(0) - 1{,}5 = 7{,}5 > 0 \Rightarrow a = 1 = y_d$ P. MAJ

* $x_1 = -2, x_2 = -2, y_d = 1$

• $z_4^0 = -3(-2) - 1{,}5(-2) - 1{,}5 = 7{,}5 > 0 \Rightarrow a = 1 = y_d$ P MAJ

Iter 1 * $x_1 = 1, x_2 = 1, y_d = 0$

• $z_1^1 = -3(1) - 1{,}5(1) - 1{,}5 = -6 < 0 \Rightarrow a = 0 = y_d$ P MAJ

* $x_1 = -1, x_2 = 1, y_d = 0$

• $z_1^1 = -3(-1) - 1{,}5(1) - 1{,}5 = 0 \geqslant 0 \Rightarrow a = 1 \neq y_d$ MAJ

$w_1^3 = -3 - 1(1-0) = -4$

$w_2^3 = -1{,}5 - 1(1-0) = -2{,}5$

$b^2 = -1{,}5 - 1(1-0) = -2{,}5$

(1)

* $x_1 = -3, x_2 = 0, y_d = 1$

• $z_3^1 = -4(-3) - 2{,}5(0) - 2{,}5 = 9{,}5 > 0 \Rightarrow a = 1 = y_d$ P MAJ

* $x_1 = -2, x_2 = -2, y_d = 1$

• $z_4^1 = -4(-2) - 2{,}5(-2) - 2{,}5 = 10{,}5 > 0 \Rightarrow a = 1 = y_d$ P MAJ

Iter 2 * $x_1 = 1, x_2 = 1, y_d = 0$

• $z_1^2 = -4(1) - 2{,}5(1) - 2{,}5 = -9 < 0 \Rightarrow a = 0 = y_d \Rightarrow$ P MAJ

* $x_1 = -1,\ x_2 = 1\ ,\ y_d = 0$

• $z_2^2 = -4(-1) - 2,5(1) - 2,5 = -1 < 0 \Rightarrow a = 0 = y_d$ PHAT

* $x_1 = -3,\ x_2 = 0\ ,\ y_d = 1$

• $z_3^2 = -4(-3) - 2,5(0) - 2,5 = 9,5 > 0 \Rightarrow a = 1 = y_d$ PHAT

* $x_1 = -2,\ x_2 = -2,\ y_d = 1$

• $z_4^2 = -4(-2) - 2,5(-2) - 2,5 = 10,5 > 0 \Rightarrow a = 1 = y_d$ PHAT

donc
$w_1^f = -4$
$w_2^f = -2,5$
$b^f = -2,5$

1

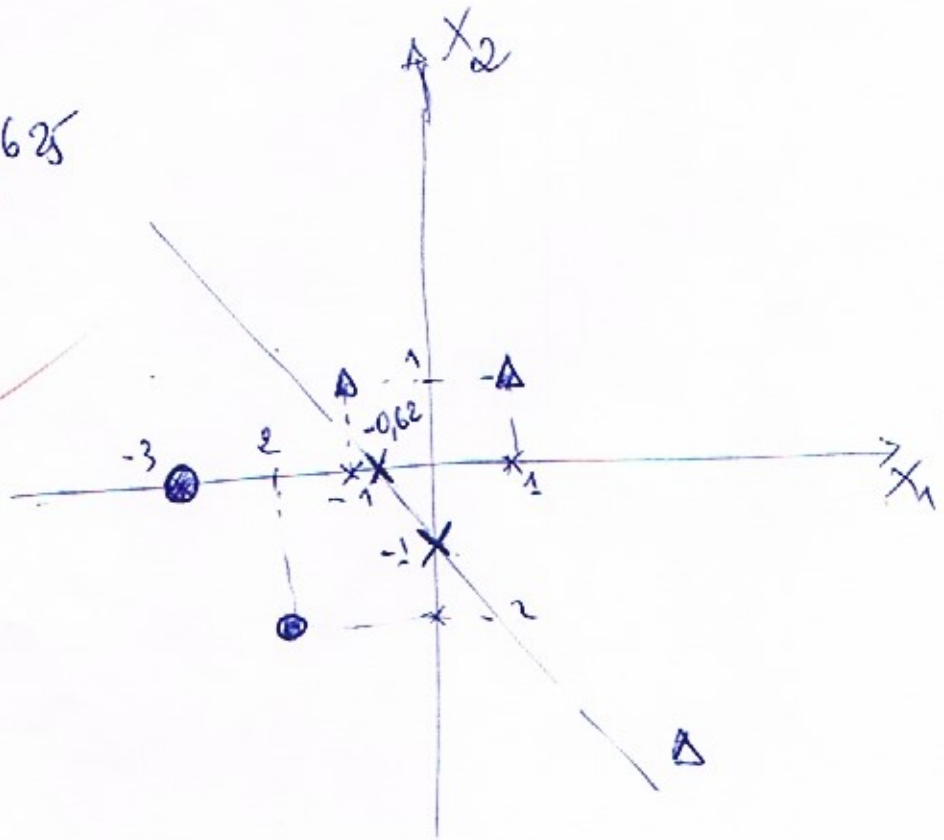
Y°/ la droite de séparation Δ correspond à l'éq

$\Delta = w_1^f x_1 + w_2^f x_2 + b^f = -4x_1 - 2,5x_2 - 2,5 = 0$

$x_1 = 0 \Rightarrow x_2 = -1$

$x_2 = 0 \Rightarrow x_1 = -2,5/4 = -0,625$

1,5



Contrôle O.I.A M2 MRI 23/24
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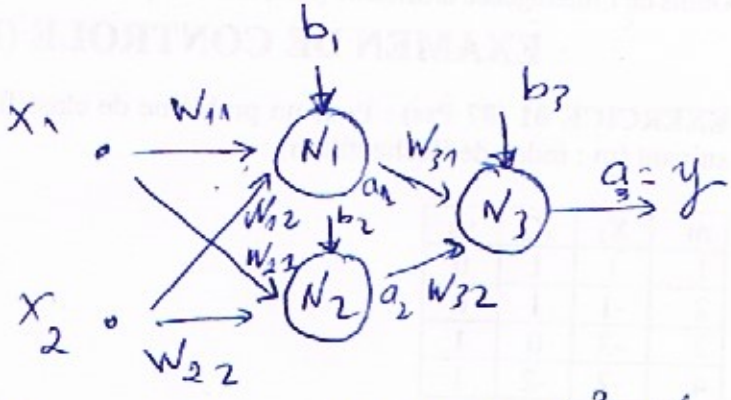
Exo 2.



1°] Eqs du Modèle : (on a $E = \frac{1}{2}(y - y_d)^2 = \frac{1}{2}(a_3 - y_d)^2$

- $z_1 = W_{11} x_1 + W_{12} x_2 + b_1 =$
- $a_1 = \varphi_1(z_1) = \frac{1 + e^{-2z_1}}{1 + e^{-2z_1}}$
- $z_2 = W_{21} x_1 + W_{22} x_2 + b_2$
- $a_2 = \varphi_2(z_2) = \frac{1 - e^{-2z_2}}{1 + e^{-2z_2}}$
- $z_3 = W_{31} a_1 + W_{32} a_2 + b_3$
- $y = a_3 = \varphi_3(z_3) = \frac{1}{1 + e^{-z_3}}$

4,5 = (0,75 x 6)

$$\begin{cases} W_{11}^1 = W_{11}^0 - \alpha \frac{\partial E}{\partial W_{11}} \\ b_2^1 = b_2^0 - \alpha \frac{\partial E}{\partial b_2} \end{cases}$$ (1) eqs de Mise à Jour par Rétropropagation du Gradient

Evaluation des Gradients :

$\star \frac{\partial E}{\partial W_{11}} = \frac{\partial E}{\partial a_3} \cdot \frac{\partial a_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial a_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial W_{11}}$ (1)

- $\frac{\partial z_1}{\partial W_{11}} = x_1$
- $\frac{\partial z_3}{\partial a_1} = W_{31}$
- $\frac{\partial E}{\partial a_3} = (a_3 - y_d)$
- $\frac{\partial a_3}{\partial z_3} = a_3(1 - a_3)$
- $\frac{\partial a_1}{\partial z_1} = 1 - a_1^2$

(1)

$$\frac{\partial E}{\partial w_{11}} = w_{31}(1-a_1^2)\,a_3(1-a_3)(a_3-y_d)\,x_1$$ ✗ (1)

$$* \frac{\partial E}{\partial b_2} = \frac{\partial E}{\partial a_3}\cdot\frac{\partial a_3}{\partial z_3}\cdot\frac{\partial z_3}{\partial a_2}\cdot\frac{\partial a_2}{\partial z_2}\cdot\frac{\partial z_2}{\partial b_2}$$ (1)

- $\frac{\partial E}{\partial a_3} = a_3 - y_d$
- $\frac{\partial a_3}{\partial z_3} = a_3(1-a_3)$
- $\frac{\partial z_3}{\partial a_2} = w_{32}$
- $\frac{\partial a_2}{\partial z_2} = 1 - a_2^2$
- $\frac{\partial z_2}{\partial b_2} = 1$

(1)

$$\Rightarrow \boxed{\frac{\partial E}{\partial b_2} = w_{32}(1-a_2^2)\,a_3(1-a_3)(a_3-y_d)}$$ 0,25 ✗ 1

$$\rightarrow w_{11}^1 = w_{11}^0 - \alpha\, w_{31}^0\, a_3(1-a_3)(1-a_1^2)(a_3-y_d)\,x_1$$

$$b_2^1 = b_2^0 - \alpha\, w_{32}^0\, a_3(1-a_3)(1-a_2^2)(a_3-y_d)$$

- $z_1 = 0,6\,(-1) - 0,1\cdot 1 + 0,3 = -0,4$
- $a_1 = \frac{1-e^{-2z_1}}{1+e^{-2z_1}} = -0,38$
- $z_2 = -0,3\,(-1) + 0,4\,(1) + 0,5 = 1,2$
- $a_2 = \frac{1-e^{-2z_2}}{1+e^{-2z_2}} = 0,83$
- $z_3 = 0,4\,(-0,38) + 0,1\,(0,83) * 0,2 = -0,27$
- $a_3 = \frac{1}{1+e^{z_3}} = 0,43$

$$\rightarrow w_{11}^1 = 0,6 - 1\cdot 0,4\cdot 0,43\cdot(1-0,43)(1-0,83^2)(0,43-1)\cdot(-1)$$

$$\boxed{w_{11}^1 = 0,582}$$ 0,25

$\rightsquigarrow b_2^1 = 0{,}5 - 1 \cdot 0{,}1 \cdot 0{,}43 (1 - 0{,}43)(1 - 0{,}93^2)(0{,}43 - 1)$

$\boxed{b_2^1 = 0{,}504}$ 0,25

2°] $\dfrac{\partial E}{\partial w_{12}} = w_{31}(1 - a_1^2)\, a_3(1 - a_3)(a_3 - y_1)\, x_2$

$\dfrac{\partial E}{\partial w_{21}} = w_{32}\, a_3(1 - a_3)(1 - a_2^2)(a_3 - y_1)\, x_1$ (1)

$\dfrac{\partial E}{\partial w_{22}} = w_{32}\, a_3(1 - a_3)(1 - a_2^2)(a_3 - y_1)\, x_1$